

Problem Set 9.8

$$\text{(Taylor series for } f(x) \text{ at } a) \quad f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

1. Find the Taylor series for $\sin x$ at $a = \frac{\pi}{3}$.

$$\text{(Maclaurin series for } f(x)) \quad f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

2. Find the Maclaurin series for $e^x \cos x$ using the Maclaurin series for e^x and $\cos x$.

3. Find the Maclaurin series for $\sin x - x + \frac{x^3}{3!}$.

(Binomial series)

$$(1+x)^p = 1 + \binom{p}{1}x + \binom{p}{2}x^2 + \binom{p}{3}x^3 + \dots,$$

where p is real number and $|x| < 1$.

4. Find the Maclaurin series for $1+x^2$ using the Binomial series.

Problem Set 9.9

(Taylor polynomial of order n based at a)

$$T_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k$$

(Remainder or error for Taylor series based at a)

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1},$$

where $f(x) = P_n(x) + R_n(x)$ and c is some point between x and a .

(Maclaurin polynomial of order n)

$$P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(0)}{k!} x^k$$

5. Find the Taylor polynomial of order 2 based at $a=1$ for $f(x) = \ln(x+2)$.

6. Find the Maclaurin polynomial of order 3 for $f(x) = e^{-3x}$.

7. Consider $f(x) = \sqrt{x}$ to approximate $\sqrt{1.1}$.

(1) Find $P_2(x)$ based at $a=1$ for $f(x)$.

(2) Use $P_2(x)$ to approximate $\sqrt{1.1}$.

8. Consider $f(x) = \sin x$ to approximate $\sin 48^\circ$.

(1) Find $P_3(x)$ based at $a = \frac{\pi}{4}$ for $f(x)$.

(2) Use $P_3(x)$ to approximate $\sin 48^\circ$.

(3) Give a good bound for the error of the approximation.